# Alfvén Waves and Structures: What Can we Learn with Multipoint Measurements on CLUSTER II

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**Abstract**. We discuss the importance of EFW and FGM instruments on CLUSTER-II for studying Alfvén waves and structures in the magnetosphere. The low frequency electromagnetic waves and structures are important for magnetosphere-ionosphere coupling, transfer of energy from the solar wind into the magnetosphere and for a wealth of auroral phenomena. The ambiguity in distinction between spatial and temporal variations are especially acute for low frequency electromagnetic phenomena related to Alfvén waves. Using measurements from FREJA and POLAR satellites we discuss a number of key unsolved problems that can be addressed with the multipoint capabilities of the CLUSTER II mission. We point out that the scale sizes of plasma and current structures in the magnetosphere can be well below 100 km, which requires a small separation distances between the four CLUSTER spacecraft.

## I. INTRODUCTION

Alfvén waves are low-frequency, electromagnetic waves in a conducting fluid with a background magnetic field. In the Alfvén wave the background magnetic field tension provides the restoring force, whereas the ion mass provides the inertia. The electric and magnetic field perturbation vectors are highly correlated in an Alfvén wave and represent two sides of the same phenomenon. Thus, the electric field experiment EFW [1] and the fluxgate magnetometer FGM [2] on CLUSTER II will play a leading role in studies of Alfvén waves in the magnetosphere.

For plane waves at frequency below the ion gyrofrequency, $\omega_{ci} = q_i B_0/m_i$, the linear analysis of MHD equations leads to three wave modes. The first one is the Alfvén wave which has the frequency

$$\omega = k_z v_A, \tag{1}$$

where

$$v_A = \frac{B_0}{(\mu_0 \rho)^{1/2}}, \tag{2}$$

is the Alfvén velocity, $k_z = k\cos\theta$, and $\rho = n_i m_i$ is the mass density. In a wave of this type there are no fluctuations of the density and the pressure, i.e., $\delta n = 0$ and $\delta p = 0$. The perturbation vector $\mathbf{b}$ is perpendicular to the planes of the vectors $\mathbf{B}_0$ and $\mathbf{k}$; the electric field $\mathbf{E}$ is perpendicular to $\mathbf{B}_0$ and lies in the $\mathbf{B}_0$, $\mathbf{k}$ plane. The fluid velocity $\mathbf{u}$ is connected with the perturbation of the magnetic field $\mathbf{b}$ by $\mathbf{u} = \mp\mathbf{b}/(\mu_0\rho)^{1/2}$, where the upper (lower) sign refers to the case $\mathbf{k}\cdot\mathbf{B}_0 > 0$ ($\mathbf{k}\cdot\mathbf{B}_0 < 0$). The other two modes are fast and slow magnetosonic modes.

The parallel electric field is important for electron acceleration and for magnetosphere-ionosphere coupling. The magnetic field-aligned electric field is produced by low-frequency ($\omega < \omega_{ci}$) dispersive Alfvén waves when the wavelength perpendicular to the background magnetic field becomes comparable either to the ion gyroradius at electron temperature, $\rho_s = (T_e/m_i)^{1/2}/\omega_{ci}$, the ion thermal gyroradius, $\rho_i = (T_i/m_i)^{1/2}/\omega_{ci}$ [3], or to the collisionless electron skin depth $\lambda_e = c/\omega_{pe}$ [4].

A recent review by Stasiewicz et al. [5] provides a comprehensive discussion on dispersive Alfvén waves in the ionospheric and laboratory plasmas. In standard terminology, the inertial Alfvén Waves (IAW) are $\omega < \omega_{ci}$ Alfvén waves in a medium where the electron thermal velocity, $v_{te} = (2T_e/m_e)^{1/2}$, is less than $v_A$. In such a case, the parallel electric field is supported by the electron inertia. Kinetic Alfvén Waves (KAW) are waves in a medium where $v_{te} > v_A$. In this case, the parallel electric force is balanced by the parallel electron pressure gradient. The term Dispersive Alfvén Waves (DAW) would cover these two cases. Clearly, the IAW arises in a low-beta plasma with $\beta = 2\mu_0 nT/B_0^2 < m_e/m_i$, whereas the KAW appear in an intermediate beta plasma with $m_e/m_i < \beta < 1$. Here, $T = (T_e+T_i)/2$ is the plasma temperature. The dispersive properties of the waves become increasingly important when the perpendicular wavelength, or characteristic scale of spatial inhomogeneities become comparable with $\rho_s$, $\rho_i$, or $\lambda_e$ which can be seen in the dispersion equation [6,5]

$$\omega = k_\parallel v_A \sqrt{\frac{1 + k_\perp^2 \rho^2}{1 + k_\perp^2 \lambda_e^2}}, \tag{3}$$

where $\rho^2 = \rho_s^2 + \rho_i^2$. We would like to emphasize that Alfvén waves, and particularly DAW, play an important role in space electrodynamics for several reasons. They are responsible for, or related to:

1. Energy transport in the form of the Poynting flux $\mathbf{S} = \mu_0^{-1}\mathbf{E}\times\mathbf{B}$ over large distances in space.
2. Global oscillations and resonances.
3. Mutual magnetosphere–ionosphere coupling.
4. Turbulence, energy cascading over a wide range of spatial scales.
5. Structuring, filamentation of plasma, creation of vortices and solitons; chaos and self-organization.
6. Ponderomotive effects.

7. Transverse ion energization and bulk heating.
8. Parallel electric field, the electron acceleration, and the formation of discrete aurora.

## II. INERTIAL ALFVÉN WAVES OBSERVED ON FREJA

FREJA was a low altitude satellite with an apogee of 1700 km, much lower than the planned perigee for CLUSTER. Nevertheless the magnetic structures to be measured by CLUSTER at the perigee should bear some resemblance to those studied by many satellites in the auroral acceleration region. The main difference is that at altitudes lower than $\sim 1R_E$ the Alfvén velocity is generally greater than the electron thermal speed and the Alfvén waves are of inertial type, whereas at higher altitudes the Alfvénic structures would have properties of kinetic waves. Also, at larger distances from the Earth, the larger value for the ion gyroradius, the electron inertial length, as well as increasing cross-section of the magnetic flux tubes would result in larger spatial scales for Alfvénic structures.

The easiest way to identify Alfvénic activity in the Freja data is through a filtered $f \geq 1$ Hz component of the magnetic field, where electromagnetic fluctuations with amplitudes of a few to several tens of nT show widespread magnetic turbulence [7,8]. The cusp and dayside cleft are found to be regions of almost permanent Alfvénic activity, while its appearance in the nightside auroral oval is closely related to periods of auroral activity. The fluctuating fields can be identified as Alfvén wave turbulence (AWT) in the frequency range $\sim 1-7$ Hz because the observed ratio $\delta E/\delta B$ is about the local Alfvén velocity $v_A$.

Figure 1 shows an example of a large region of Alfvén turbulence observed in the cleft region. One can notice a general correlation between large scale density depressions and Alfvén wave intensity (Fig. 1c) which indicates that Alfvén waves play a major role in heating and evacuating the plasma along the magnetic field lines.

The Fourier spectrum of the data shown in Figure 1a,b would represent broadband spectrum up to a few hundred Hz frequently referred to as broadband ELF waves. On the other hand, the Alfvén mode is expected to have a frequency much lower than the ion cyclotron frequency ($f_{cO+} \sim 25$ Hz in Freja environment). There are indications that the Fourier spectrum of electromagnetic signals around or below 1 Hz corresponds to Alfvén wave frequencies. Thus, the broadband waves above the oxygen gyrofrequency cannot be associated to Alfvén turbulence in the time domain. The nature of these waves is not obvious and represent a challenging problem for space measurements because of the difficulty in identifying spatial and temporal variations with measurements on board a single spacecraft. As has been discussed by Stasiewicz et al. [9], the presence of short-length dispersive Alfvén waves in the regions of AWT produces a Doppler shifted spectrum of waves at higher frequencies, equivalent to broadband ELF waves.

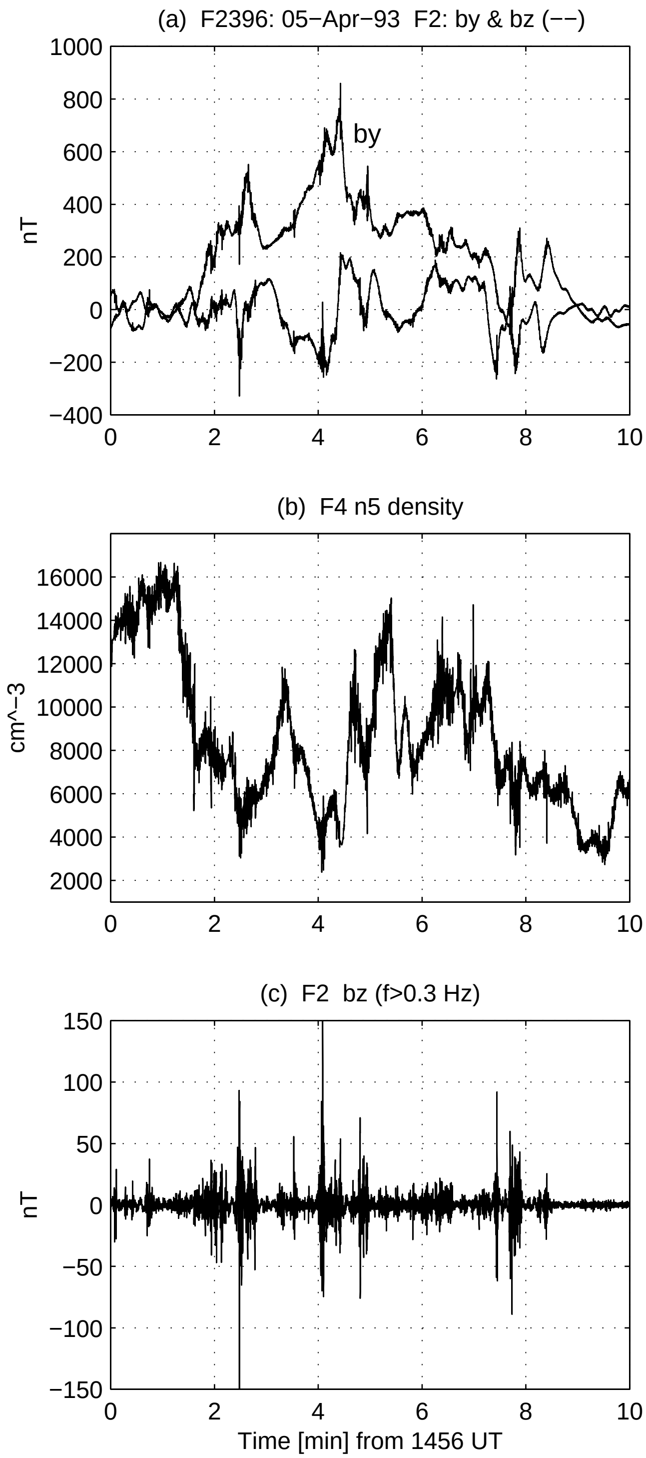


FIG. 1. A large region of Alfvén turbulence in the post-afternoon region (a) perpendicular components of the magnetic field, (b) plasma density derived from the Langmuir probe, (c) AC component of the magnetic field (from Ref. [8]).

For example, low frequency Alfvén waves, $\omega < \omega_{ci}$, with perpendicular wavelengths $\sim 10$ m up to $\sim 7$ km would be Doppler shifted ($f = V_s/\lambda$) in the spacecraft reference frame and observed as waves in the frequency range $\sim 1-700$ Hz. The ratio $\delta E/\delta B$ can be used to investigate properties of the low frequency turbulence. In the cold electron limit and finite ion gyroradius plasma, the ratio is given by [9]

$$\left|\frac{E_\perp}{B_\perp}\right| = v_A\sqrt{(1+k_\perp^2\lambda_e^2)(1+k_\perp^2\rho_i^2)}. \qquad (4)$$

Thus, the electric to the magnetic field ratio for inertial waves with short perpendicular wavelengths can be much larger than the local Alfvén speed. An analysis of several events measured on Freja [9] shows that the spectrum of Alfvén wave turbulence is very well represented by relation (4).

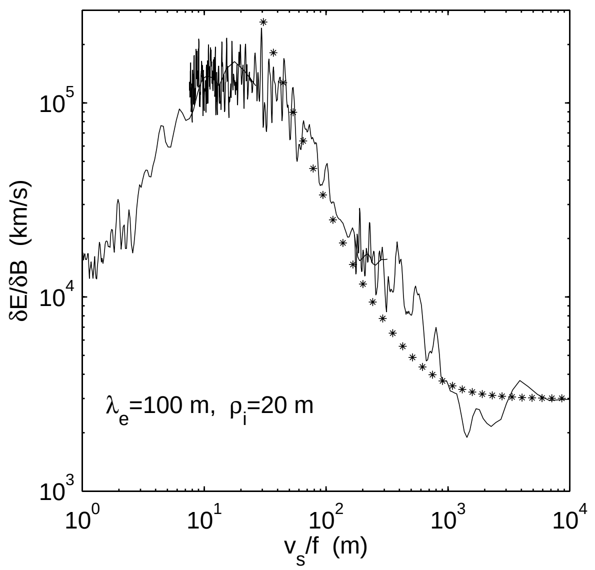


FIG. 2. $\delta E/\delta B$ ratio for a 15 s time interval of FREJA data discussed in Ref. [9]. The fit is made with the dispersion given by (4). A single dispersive Alfvén mode is seen at $\lambda = v_s/f \sim 30-10,000$ m. Parameters: $v_A = 3,000$ km/s, $\lambda_e = 100$ m, and $\rho_i = 20$ m.

This interpretation is based on the assumption that the observed variations are mainly spatial and not temporal. Since there would be no reason for arbitrary broadband waves in this frequency domain to lie just on DAW dispersion relation (4), it is deduced that the low-frequency part (below 500 Hz) of broadband ELF waves represents mainly spatial turbulence of DAW. Below $\lambda = v_s/f \approx 30$ m we see that finite antenna length effects and/or finite ion gyroradius effects [9] determine the character of the $\delta E/\delta B$ ratio.

## III. MULTIPOINT DENSITY MEASUREMENTS ON A SINGLE SPACECRAFT

The EFW experiment on CLUSTER can be run in a "density" mode where two or four spherical probes mounted on the tips of 50 m wire booms can measure current from the surrounding plasma instead of the potential difference. When the probe is biased to collect the electron current, one can use the following approximation for the probe current

$$j_e \propto en(T_e/m_e)^{1/2}(1+V_p/T_e), \qquad (5)$$

where $V_p$ is the probe potential and $T_e$ is the electron temperature. The current depends mainly on the electron density and temperature. EFW working in this mode is unable to deliver the electric field signal for higher frequency analysis by other instruments within the WEC consortium [10]. For this reason use of the EFW density mode has to be limited. However, we would like to emphasize unique capabilities of this mode for providing multipoint measurements on single spacecraft. Such capabilities are very useful for distinguishing between temporal and spatial variations at small scales. This point is illustrated by Figure 3 which shows a case from FREJA.

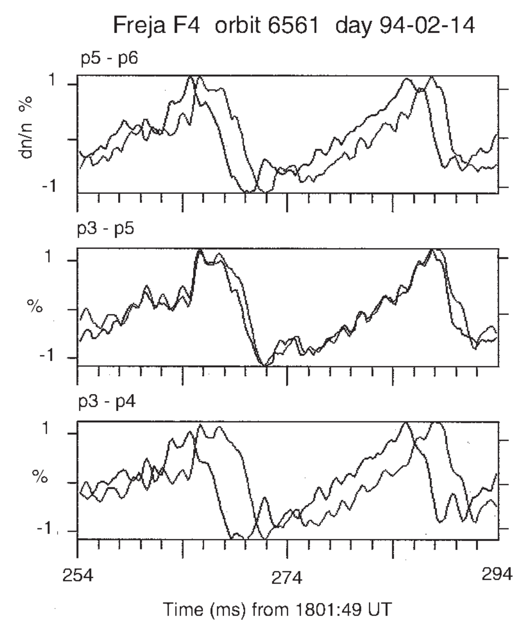


FIG. 3. Density variations measured on FREJA by four density probes mounted on wire booms of 20 m. The time shift of signals measured by different probes (p3, p4, p5, p6) is consistent with spatial density structures crossed by the satellite moving with 6.5 km/s.

Two curves on each panel correspond to the density variation measured by a pair of the density probes, which have different orientation in respect to the magnetic field and thus different baseline length perpendicular to the magnetic field. Visual inspection shows a time shift between two records which is consistent with a density structure which is stationary, or slowly drifting through the plasma.

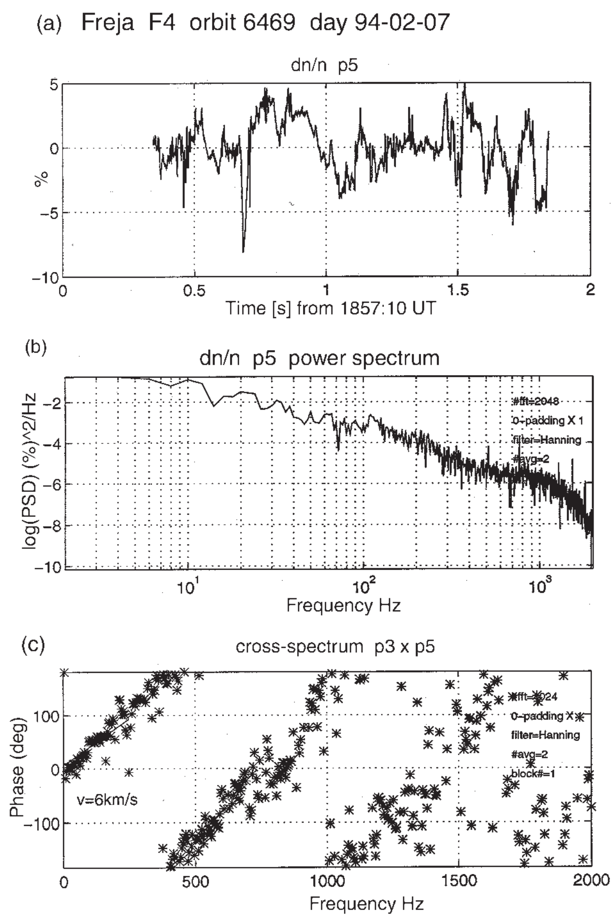


FIG. 4. Density fluctuations (a), power spectrum (b) and cross-spectrum (c) of two density signals for a case from FREJA (orbit 6469 94-02-07).

The visual analysis of signals and determination of the time-lag is feasible for macroscopic structures. For smaller scales, or higher frequencies, the visual technique is unsatisfactory and one has to apply spectral methods. Let us consider a density wave $\delta n \propto \cos(k_\perp x - \omega t)$ propagating perpendicularly to $\mathbf{B}_0$, and along the antenna direction. The phase difference between two density probes separated by a distance $d$ is $\Delta\phi = k_\perp d$. Introducing the Doppler relation $\mathbf{v}\cdot\mathbf{k} = \omega$ into this expression, we get

$$\Delta\phi = 2\pi d f / v. \quad (6)$$

Thus, one expects a linear dependence of the phase on the frequency. The velocity of the structure with respect to the satellite can be determined from the slope of the linear relation. With cross-spectral analysis of two density signals we can determine the phase shift, the coherency of two signals, and the dependence $\Delta\phi(f)$ [11]. Figure 4 shows the density fluctuations (a), power spectrum (b) and the cross-spectrum in Figure 4c. The power spectrum shows broadband (BB-ELF) waves extending up to 1 kHz with a power-law distribution. The phase of the cross-spectrum is shown as a function of frequency. The measured phase shift between the two signals is linear up to ~800 Hz and the velocity of the structures derived from the slope of the linear relation is close to the satellite speed. This is consistent with the interpretation that structures down to 10 m (up to 800 Hz in the satellite reference frame) are spatial, or very slowly moving ($v_\perp \ll v_s$). Thus, the measurements indicate that the apparent wave frequency in fact is not in the time domain, but corresponds to Doppler shifted small scale density structures.

The above described technique can be used also on CLUSTER for solving spatio-temporal ambiguities for small scale solitary structures. Use of this technique requires that the EFW instrument is run in the density mode for at least two spherical sensors. The EFW instrument is capable of sampling two signals up to 36 ksamples/s.

## IV. ELECTROMAGNETIC STRUCTURES MEASURED BY POLAR

The results from the POLAR satellite (apogee of $9R_E$) are very relevant for CLUSTER science. We shall show here examples of electromagnetic structures measured by POLAR in the cusp region. Similar structures are expected to be also encountered by CLUSTER-II.

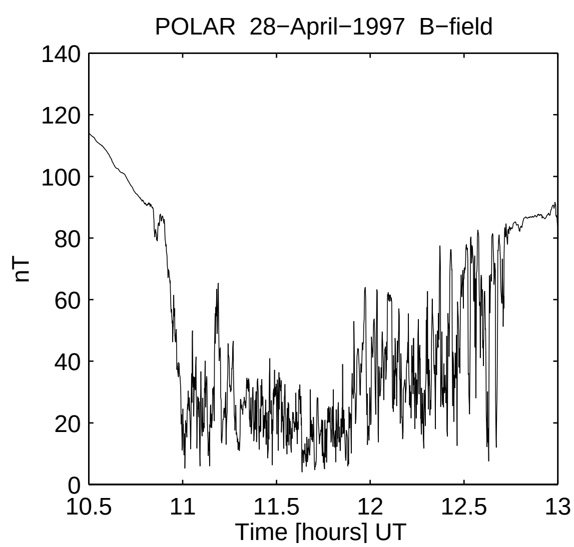


FIG. 5. Total magnetic field measured by POLAR on April 28, 1997. It shows structured magnetic depression detected in the cusp region at the geocentric distance of $7R_E$.

Figure 5 shows the total magnetic field measured by POLAR in the cusp region on April 28, 1997. A very structured magnetic depression is seen during 1100-1240 UT. This type of structures is often observed by the HAWKEYE, POLAR and INTERBALL spacecraft in the vicinity of the outer cusp region [12,13]. By using the electric field measurements (not shown here) one can demonstrate that the observed fluctuations are of Alfvénic type. The magnetic and electric field fluctuations are highly correlated and consistent with the dispersion relation (4) for dispersive Alfvén waves.

As discussed earlier, it is difficult to estimate spatial scale of plasma structures with a single spacecraft. The CLUSTER fleet of spacecraft is expected to provide significant contributions for resolving this issue. At lower altitudes, in the FREJA case, the convective plasma flows are typically less than 1 km/s and thus much smaller than the satellite velocity of 7 km/s. The spatial structures are thus fast traversed by the spacecraft. A different situation is encountered by POLAR at the apogee, where the convective plasma flows may exceed 50 km/s, and thus much faster than the satellite speed of 3 km/s. To see what we may expect to find on CLUSTER in the cusp region we focus on 3 minutes time interval from Figure 5. The record of the axial component of the magnetic field, sampled at the rate of 8 Hz is displayed in Figure 6. It shows multiscale structures observed during intervals of the order of seconds.

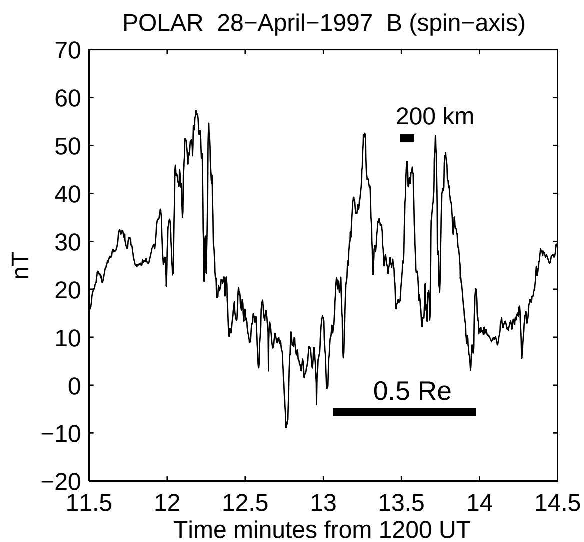


FIG. 6. High-resolution magnetic field records within the time interval of Figure 5. The POLAR measurements indicate that the spatial scales encountered in the cusp may be less than 100 km, much smaller than the nominal planned CLUSTER separation distance. The spatial dimensions are estimated with a convection velocity of 50 km/s.

Using an average convection velocity of 50 km/s we find that a structure observed during 4 s would have a size of 200 km and a structure observed during 1 min would have a dimension of 3000 km $\approx 0.5R_E$. These intervals are marked on the plot in Figure 6. It is clear that if the spacecraft separation is larger than the size of the magnetic structures, one cannot use the four spacecraft for the determination of curl **B** and for derivation of macroscopic gradients. It is therefore advisable to keep the spacecraft separation as small as technically possible.

## V. SUMMARY

In his paper we have discussed the importance of EFW and FGM instruments for studying Alfvén waves and structures in the magnetosphere. Alfvén wave related phenomena are of key importance for solar wind - magnetosphere coupling, for magnetosphere - ionosphere interactions and for a wealth of auroral phenomena. The multipoint capabilities of the CLUSTER fleet will help to remove the spatio-temporal ambiguity inherent in space measurements of low frequency Alfvénic phenomena. Using examples from the FREJA spacecraft we have shown that spatial structures encountered in the topside ionosphere are frequently less than 1 km width, perpendicular to the magnetic field. At higher (POLAR) altitudes in the cusp and in the magnetopause boundary layer the electromagnetic structures may be smaller than 100 km. In order to properly resolve properties of such structures a comparable spacecraft separation is required. Another class of extremely narrow (~kilometers scale) solitary structures and sharp gradients cannot be resolved even at a very small spacecraft separation. In such cases the multipoint capabilities of the EFW instrument working in the density mode can be exploited.

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